Variations in Solar Luminosity from Time Scales of Minutes to Months

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ABSTRACT

We present the power spectrum of solar irradiance during 1985 and 1987 obtained from the ACRIM project from time scales of minutes to months. At low frequency the spectra are Lorentzian (proportional to $1/(f^2 + f_o^2)$). At higher frequencies they are proportional to $f^{-\frac{1}{2}}$. A linear, stochastic model of the turbulent heat transfer between the granulation layer (modeled as a homogeneous thin layer with a radiative boundary condition) and the rest of the convection zone (modeled as a homogeneous thick layer with thermal and diffusion constants appropriate the lower convection zone) predicts the observed spectrum.

Subject headings: sun: activity, turbulence, convection, diffusion

1. Introduction

The luminosity of the sun has significant variations on time scales from minutes to years. Data from the NIMBUS 7 and ACRIM projects have provided us with a high quality time series of these variations. Some aspects of these variations can be attributed to specific physical processes (see Stix 1989 for an introductory review). For example, at very high frequencies, oscillations of the sun result in a well-understood five minute periodicity. At the yearly and decadal time scale there is a strong correlation between the irradiance and variations in the solar magnetic activity. Variations from minutes to months have a simple spectral form but are not well understood (Frohlich 1993).

Frohlich (1987) has published power spectra of ACRIM data from 1980 and 1985. He reported that the power spectrum is flat for frequencies corresponding to time scales greater than one week, proportional to f^{-2} for time scales between twelve hours and one week, and proportional to f^{-1} at time scales down to minutes. Our studies agree that the low frequency spectrum is Lorentzian. We find, however, that the high-frequency spectrum is proportional to $f^{-\frac{1}{2}}$ at time scales shorter than one day.

Kuhn, Libbrecht, & Dicke (1988) have suggested the possibility that the interaction between the solar surface and deeper portions of the convection zone may cause the low-frequency variations. We explore that possibility in this *Letter*.

If turbulent transfer in a system is dominated by eddies much smaller than the system size, random convective action of turbulent eddies will be analogous to the molecular agitation responsible for molecular diffusion (Moffatt 1983). In that approximation, turbulent transfer can be modeled as a stochastic diffusion process. Random forcing of a deterministic diffusion equation enables analytic study of the fluctuations from equilibrium resulting from the stochasticity of turbulent transfer in any geometry with

any linear boundary condition. In this paper we study the fluctuations in luminosity of a thin homogeneous surface granulation layer with a linear radiation boundary condition exchanging heat with a deep, homogeneous layer below with density and diffusion constants appropriate to the lower convection zone. The high and low crossover frequencies in the spectrum correspond to time scales of thermal and radiative equilibration of the convection zone, respectively. The time scales for equilibration are given by the model as a function of thermal and diffusion constants of the granulation layer and lower convection zone. Estimates of these constants with mixing length theory yields order-of-magnitude agreement between the crossover frequencies predicted by the model and those observed.

2. Power Spectrum of ACRIM Data

In Figure 1 we present the logarithm of the normalized Lomb periodograms of ACRIM solar irradiance data sampled during 1987 and 1985 plotted as a function of the logarithm of the frequency. We chose to analyze these years since they appear to represent extremes in the variation of solar activity at low frequencies. The low-frequency variance in the 1985 and 1987 data are small and large, respectively. Variations in the solar irradiance at yearly time scales are generally agreed to be the result of variations in magnetic activity. We have chosen these years in order to assess the influence of magnetic activity on the power spectrum and distinguish its influence from that of the mechanism proposed in our model.

Since the data were sampled at irregular intervals, simple FFT methods of estimating the power spectrum are only available if we average the data over some uniform time interval. We chose instead to use the Lomb periodogram suggested by Press et al. (1992) for unevenly sampled data. Above frequencies of $\log f = -2.0$ we averaged the periodogram in logarithmically spaced frequency intervals of $\log f = 0.01$ to reduce the scatter. We

subtracted $\log S(f)$ by 2.5 to plot it on the same graph as the 1987 data.

The high-frequency behavior is the same for both spectra. A f^{-2} region flattens out to a $f^{-\frac{1}{2}}$ at frequencies greater than $f \approx \frac{1}{1\ day}$. Our observation of a $f^{-\frac{1}{2}}$ scaling region at high frequencies disagrees with Frohlich's (1987) conclusion that the high-frequency scaling region is proportional to f^{-1}. He reported this conclusion for 1985 ACRIM data, the same data we analyzed as part of our study. Our confidence in our interpretation lies in the greater resolution of our spectra, whose $f^{-\frac{1}{2}}$ range is 50% larger than Frohlich's (1987). Large peaks appear at the orbital frequency of the satellite and its harmonics. These peaks are an artifact of the spectral estimation.

The low-frequency behavior of both spectra are Lorentzian (constant at low frequencies and proportional to f^{-2} at higher frequencies) in agreement with Frohlich's (1987) results. Aside from the basic form of the spectra, there is a large variability between the crossover frequencies and the magnitude of the two spectra reported here and with those published by Frohlich (1987). This variability was also discussed by Frohlich (1987). The crossover frequency of the Lorentzian portion of the 1987 and 1985 data reported here are $f = \frac{1}{5\ months}$ and $f = \frac{1}{1\ month}$, respectively. We interpret this variability as due to either variations in the magnetic activity or to limitations of our model at these time scales. This *Letter* leaves this an open question.

3. Model of Variations in Solar Luminosity

The variations in the irradiance of the sun will be proportional to the variations in its surface temperature. This follows from the fact that the power emitted by the sun, modeled as a blackbody, $F - F_e = \sigma T^4 - \sigma T_e^4$, can be well approximated by a linear dependence on $T - T_e$ for small departures from equilibrium.

Turbulent transport of heat in the convection zone of the sun can be modeled by a stochastic diffusion process. A stochastic diffusion process can be studied analytically by adding a noise term to the flux of a deterministic diffusion equation (van Kampen 1981):

$$\rho c \frac{\partial \Delta T}{\partial t} = -\frac{\partial J}{\partial x} \tag{1}$$

$$J = -\sigma \frac{\partial \Delta T}{\partial x} + \eta(x, t) \tag{2}$$

where ΔT are the fluctuations in temperature from equilibrium and the mean and variance of the noise is given by

$$\langle \eta(x, t) \rangle = 0 \tag{3}$$

$$\langle \eta(x, t)\eta(x', t') \rangle \propto \sigma(x)\langle T(x) \rangle^2 \delta(x - x')\delta(t - t') \tag{4}$$

Methods of studying transport by adding a noise term to a deterministic differential equation are termed Langevin methods. Langevin methods have been widely used to study the turbulent diffusion of a passive scalar (temperature or contaminant concentration) in the Earth's atmosphere and oceans (see Csanady 1980 for a review).

A diffusion process has a frequency-dependent correlation length $\lambda = (\frac{2D}{f})^{\frac{1}{2}}$ (Voss & Clarke 1976). At very high frequencies the correlation length is smaller than the convection zone. In that scaling region, the fluctuations in the temperature of the granular layer of the sun have a power spectrum proportional to $f^{-\frac{1}{2}}$. To show this, we present an argument due to Voss & Clarke (1976). The Fourier transform of the driven diffusion equation presented above is

$$\Delta T(k, \omega) = \frac{ik\eta(k, \omega)}{Dk^2 - i\omega} \tag{5}$$

The frequency-dependent correlation function, $c_T(s, \omega) = < \Delta T(x + s, \omega)\Delta T^*(x, \omega) >$, is then given by (Voss & Clarke 1976)

$$c(s, \omega) \propto \frac{e^{-\frac{s}{\lambda}}}{\omega^{\frac{1}{2}}} \cos(\frac{\pi}{4} + \frac{s}{\lambda}) \tag{6}$$

The power spectrum of fluctuations in the average temperature in a layer of width l will be a double spatial integral of the correlation function over pairs of points in the volume of length l and a time integral of the integrand multiplied times a factor $\cos(\omega\tau)$ (the Wiener-Khintchine theorem). For low frequencies, the factor $\cos(\omega\tau)$ is one so that the power spectrum reduces to the spatial integral

$$S_T(\omega) \propto \int_0^l dx_1 \int_0^l dx_2 c_T(x_1 - x_2, \omega) \tag{7}$$

Since the correlation function is independent of $x_1 - x_2$ for low frequencies, integration will yield the same frequency dependence. Thus, $S_T(\omega) \propto \omega^{-\frac{1}{2}}$.

At lower frequencies, the entire convection zone achieves thermal equilibrium. The variance in temperature of the convection zone (and therefore the irradiance) will now be determined by the radiation boundary condition. The fluctuating heat transport (a consequence of the stochasticity of turbulence) near the surface adds and subtracts heat from the top of the convection zone. This results in temperature and irradiance variations with a random walk (f^{-2}) spectrum.

The fluctuating input and output of heat in the f^{-2} region will cause large variations from equilibrium. When the temperature of the convection zone becomes larger than the equilibrium temperature, it will radiate, on average, more heat than at equilibrium. Conversely, when the temperature of the convection zone wanders lower than the equilibrium temperature, less heat is radiated. This negative feedback limits the variance at low frequencies resulting in a constant power spectrum.

The Lorentzian portion of the spectrum has exactly the same physics as shot noise in an RC circuit. In shot noise, the Brownian motion of electrons gives rise to a f^{-2} spectrum of charge stored on the capacitor. The frequency-dependent reactance of the capacitor limits the variance of the charge through a transient response $\frac{dQ}{dt} = \frac{Q}{\tau_o}$ where $\tau_o = 2\pi RC$. This response is equivalent to the linear radiation boundary condition we have imposed at

the surface of the sun. In our model, two homogeneous layers with different thermal and diffusion constants comprise the interacting "circuit" elements. The crossover frequency is given by $f_o = \frac{1}{2\pi R_{eq} C_{eq}}$ where R_{eq} and C_{eq} are the equivalent thermal resistance (inverse of the thermal conductivity) and the thermal capacitance of the parallel combination of the two layers, respectively.

The model we present was solved in the context of a different problem by van Vliet, van der Ziel, & Schmidt (1980). They considered the temperature fluctuations in a thin metal film supported by a substrate.

The geometry of the model is a thin granulation layer of width 2×10^6 m and uniform density (equal to the density at the bottom of the granulation layer where most of the heat capacity resides) of 0.003 kg/m^3 coupled to a thick layer of uniform density representing the rest of the convection zone. The layers have a planar geometry for convenience in this simplified model. The turbulent diffusivity is estimated from mixing length theory as $\alpha = \frac{1}{3}vl$ where v and l are the characteristic velocity and eddies sizes, respectively. The eddy size, l, is usually approximated as one pressure scale height. In the case of the granulation layer, however, the dominant eddy size is the size of the convection cell, approximately 2×10^6 m. The velocity at the bottom of the granulation layer is on the order of 1000 m/s. These estimates yield an eddy diffusivity of 10^9 m^2/s near the solar surface. The thermal conductivity, $\sigma = \alpha\rho c$, is 3×10^7 W/m^{o}K since $c = 10$ J/kg^{o}K for a monatomic hydrogen gas. The velocity, thickness, and specific heat values are from a standard solar model presented in Stix (1992). The density values are from Bohm (1963).

The granulation layer sits atop the rest of the convection zone. We approximate the density, width, and diffusivity of the remainder of the convection zone by their values near the bottom of the convection zone because its high density results in a concentration of heat capacity there and its slow diffusivity is the rate-limiting step of thermal equilibration of

the convection zone. The width and dominant eddy scale will both be given by 10^7 m, the width of the lowest pressure scale height of the convection zone. The density is estimated to be 0.5 kg/m^3 and the velocity is on the order of 20 m/s. These values are the arithmetic means of the densities and velocities at the top and bottom of the pressure scale height. These values yield an eddy diffusivity of 10^8 m^2/s for the bottom of the convection zone. Our diffusivities agree with the estimates of Stix (1992) who quote the range of diffusivities in the convection zone as $10^8 - 10^9$ m^{2}s^{-1}. The thermal conductivity is 3x10^8 W/m^{o}K.

The equation for temperature fluctuations in space and time in the model is

$$\frac{\partial \Delta T(x,t)}{\partial t} - \alpha(x)\frac{\partial^2 \Delta T(x,t)}{\partial x^2} = -\frac{\partial \eta(x,t)}{\partial x} \tag{8}$$

with

$$\langle \eta(x,t) \rangle = 0 \tag{9}$$

$$\langle \eta(x,t)\eta(x',t') \rangle \propto \sigma(x)\langle T(x) \rangle^2 \delta(x-x')\delta(t-t') \tag{10}$$

The boundary conditions are that there be no heat flow out of bottom of the convection zone and continuity of temperature and heat flux at the boundary separating the granulation layer and the deeper convection zone:

$$\sigma'\frac{\partial T}{\partial x}\Big|_{x=w_2} = 0 \tag{11}$$

$$\Delta T(x=w_1^+) = \Delta T(x=w_1^-) \tag{12}$$

$$\sigma\frac{\partial \Delta T}{\partial x}\Big|_{x=w_1^-} = \sigma'\frac{\partial \Delta T}{\partial x}\Big|_{x=w_1^+} \tag{13}$$

where w_1 and w_2 are the widths of the granulation layer and deep convection zone, respectively and the primes denote thermal and diffusion constants of the deep convection zone.

At the top of the granulation layer we impose a blackbody radiation boundary condition. The heat emitted from the granulation layer is dependent on the temperature

of the layer through the Stefan-Boltzmann law. Since solar surface temperature variations are small, within a linear approximation, the emitted temperature will be proportional to the temperature difference from equilibrium (the same approximation is commonly made in analytic models of climate change, i.e. Ghil 1983). At the solar surface, then,

$$\sigma \frac{\partial \Delta T}{\partial x}\Big|_{x=0} = g \Delta T(x = 0) \tag{14}$$

where $g = 4\sigma_b T_o^3 = 2\text{x}10^4$ W/m^{2o}K is the thermal conductance of heat out of the sun and σ_b is the Stefan-Boltzmann constant.

van Vliet et al. (1980) used Green's functions to solve this model. The Green's function of the Laplace-transformed diffusion equation is defined by

$$i\omega G(x, x', i\omega) - \alpha(x)\frac{\partial^2 G(x, x', i\omega)}{\partial x^2} = \delta(x - x') \tag{15}$$

where G is governed by the same boundary conditions as ΔT. This equation can be solved by separating G into two parts: G_a and G_b with $x < x'$ and $x > x'$, respectively, where G_a and G_b satisfy the homogeneous (no forcing) diffusion equation with a jump condition relating G_a and G_b:

$$\frac{\partial G_a}{\partial x}\Big|_{x=x'} - \frac{\partial G_b}{\partial x}\Big|_{x=x'} = \frac{1}{\alpha(x')} \tag{16}$$

The power spectrum of the average temperature in the granualtion layer in terms of G is given by van Vliet et al. (1980) as:

$$S_{\Delta T_{av}}(f) \propto Re(\int_0^{w_1} \int_0^{w_1} G_1(x, x', i\omega)dxdx') \tag{17}$$

$$\propto Re(\int_0^{w_1} \int_0^{x} G_{1b}(x, x', i\omega)dxdx' + \int_0^{w_1} \int_x^{w_1} G_{1a}(x, x', i\omega)dxdx') \tag{18}$$

where G_1 stands for the solution to the differential equation for G where the source point is located in the granulation layer. Two forms of G_{1a} and G_{1b} are necessary for x located above and below x', respectively, due to the discontinuity in the derivative of G_1 created

by the delta function (the jump condition). The solution of G_1 which satisfies the above differential equation and boundary conditions is

$$G_{1a} = \frac{L}{\alpha K}(\frac{\sigma'L}{\sigma L'}\sinh(\frac{w_1 - x'}{L})\sinh(\frac{w_2}{L'}) + \cosh(\frac{w_1 - x'}{L})\cosh(\frac{w_2}{L'}))(\sinh(\frac{x}{L}) + \frac{\sigma}{Lg}\cosh(\frac{x}{L}))$$

(19)

and

$$G_{1b} = G_{1a} + \frac{L}{\alpha}\sinh(\frac{x' - x}{L})$$

(20)

where

$$K = (\sinh(\frac{w_1}{L}) + \frac{\sigma}{Lg}\cosh(\frac{w_1}{L}))\frac{\sigma'L}{\sigma L'}\sinh(\frac{w_2}{L'}) + (\cosh(\frac{w_1}{L}) + \frac{\sigma}{Lg}\sinh(\frac{w_1}{L}))\cosh(\frac{w_2}{L'})$$

(21)

and $L = (\frac{\alpha}{i\omega})^{\frac{1}{2}}$ and $L' = (\frac{\alpha'}{i\omega})^{\frac{1}{2}}$. Performing the integration van Vliet et al. obtained

$$S_{\Delta T_{av}}(f) \propto Re(L^2(\frac{\sigma'L}{\sigma L'}\tanh(\frac{w_2}{L})((\frac{gw_1}{\sigma} - 1)\tanh(\frac{w_1}{L}) - \frac{2gL}{\sigma}\frac{\cosh(w_1/L) - 1}{\cosh(w_1/L)} + \frac{w_2}{L})$$
$$+(\frac{gw_1}{\sigma} + (\frac{w_1}{L} - \frac{gL}{\sigma}\tanh(\frac{w_1}{L}))((\tanh(\frac{w_1}{L}) + \frac{\sigma L}{g})\frac{\sigma'L}{\sigma L'}\tanh(\frac{w_2}{L'}) + (1 + \frac{\sigma}{Lg}\tanh(\frac{w_1}{L})))^{-1}$$

(22)

For very low frequencies,

$$\tanh(\frac{w_1}{L}) \approx \frac{w_1}{L}, \tanh(\frac{w_2}{L'}) \approx \frac{w_2}{L'}$$

(23)

$$\frac{\cosh(w_1/L) - 1}{\cosh(w_1/L)}) \approx \frac{1}{2}\frac{w_1^2}{L^2}$$

(24)

Reducing eq. (22),

$$S_{\Delta T_{av}}(f) \propto \frac{1}{1 + \frac{\omega^2}{\omega_0^2}} \propto \frac{1}{f^2 + f_0^2}$$

(25)

which is the low-frequency Lorentzian spectrum observed in the ACRIM data. The crossover frequency as a function of the constants chosen for the model is

$$f_0 = \frac{g}{2\pi(w_1 c\rho + w_2 c'\rho'(1 + \frac{gw_1}{\sigma}))} \approx \frac{\sigma}{2\pi w_1 w_2 c'\rho'} \approx \frac{1}{8 \text{ months}}$$

(26)

which is within an order of magnitude of the observed crossover frequencies of the 1987 and 1985 ACRIM data, $f = \frac{1}{5 \text{ months}}$ and $f = \frac{1}{1 \text{ month}}$ respectively.

At low frequencies

$$\tanh(\frac{w_1}{L}) \approx \frac{w_1}{L}, \tanh(\frac{w_2}{L'}) \approx 1 \tag{27}$$

$$\frac{\cosh(w_1/L) - 1}{\cosh(w_1/L)} \approx \frac{1}{2}\frac{w_1^2}{L^2} \tag{28}$$

then

$$S_{T_{av}}(f) \propto \frac{1}{2}(\frac{2gw_1}{\sigma})^{\frac{1}{2}}(\frac{c\rho\sigma}{c'\rho'\sigma'})^{\frac{1}{2}}(\frac{g}{w_1\rho c f})^{\frac{1}{2}} \propto f^{-\frac{1}{2}} \tag{29}$$

as observed.

The high and low-frequency spectra meet at

$$f_1 = \frac{g}{w_1\rho c}(\frac{\sigma}{2gw_1})^{\frac{1}{3}}(\frac{c'\rho'\sigma'}{c\rho\sigma})^{\frac{1}{3}}4^{\frac{1}{3}}(\frac{c\rho w_1}{c'\rho' w_2})^{\frac{4}{3}} \tag{30}$$

$$\approx \frac{1}{6 \ hours} \tag{31}$$

which also agrees to within an order of magnitude with the crossover frequency of the ACRIM data ($f = \frac{1}{1 \ day}$).

We have applied the same model to climate change (Pelletier 1995). The above model gives the fluctuations in the average temperature in a thin, homogeneous layer (Earth's atmosphere) coupled with another homogeneous layer with different thermal and diffusion constants (the ocean). As part of that work, we analyzed the Vostok ice core. We found the same spectral form reported here in the ACRIM data. The thermal and radiative time scales in that dataset were $2,000$ and $40,000$ years, respectively. Estimates of those time scales based upon thermal constants and diffusion constants inferred from tracer studies in the atmosphere and ocean matched the time scales of the crossover frequencies of the Vostok data well.

4. Conclusions

We have presented evidence that the power spectrum of variations in solar irradiance exhibits four scaling regions. We presented a model originally due to van Vliet et al. (1980) proposed to study temperature fluctuations in a metallic film (granulation layer) supported by a substrate (deep convection zone) that matches the observed frequency dependence of the power spectrum of irradiance fluctuations.

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Fig. 1.— Figure 1: Logarithm of the normalized Lomb periodograms of solar irradiance in 1987 (upper plot) and 1985 (lower plot) from the ACRIM project versus the logarithm of the frequency in hours^{-1}. The crossover frequencies for 1987 are $f_0 = \frac{1}{5\ months}$ and $f_1 = \frac{1}{1\ day}$. The crossover frequencies for 1985 are $f_0 = \frac{1}{1\ month}$ and $f_1 = \frac{1}{1\ day}$. The 1985 spectrum is shifted down by $\log S(f) = -2.5$.